

MATH 504 HOMEWORK 3

Due Monday, February 29.

Problem 1. *Suppose that κ is strongly inaccessible. Show that:*

- (1) α is an ordinal iff $V_\kappa \models$ “ α is an ordinal”.
- (2) α is a cardinal iff $V_\kappa \models$ “ α is a cardinal”.
- (3) α is a regular cardinal iff $V_\kappa \models$ “ α is a regular cardinal”.
- (4) α is strongly inaccessible iff $V_\kappa \models$ “ α is strongly inaccessible”.

Note: the above problem shows that if κ is the least inaccessible cardinal, then $V_\kappa \models$ “there are no inaccessible cardinals”. It follows that it cannot be proved in ZFC that inaccessible cardinals exist.

Problem 2. *Assume that $V = L$. Prove that $V_\alpha = L_\alpha$ iff $\alpha = \aleph_\alpha$. Here you can use the theorem that in $V = L$, GCH holds.*

Problem 3. *Suppose that κ is a regular uncountable cardinal in L . Recall that we showed in class that L_κ also satisfies Comprehension. Show that L_κ satisfies the rest of the axioms of $ZF \setminus$ Powerset.*

Problem 4. *Suppose that $M \prec L_{\omega_1}$. Show that M is transitive. (Hint: for $X \in M$, take the \prec_L -least onto $f : \omega \rightarrow X$. Show that f is definable in L_{ω_1} from X and use this to show that $f \in M$. Also show $\omega \subset M$. Use these to prove that range of f is a subset of M)*